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Stochastic Resonance on FitzHugh-Nagumo System to Realize Perceptual Functions in Noisy Environment

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Edge detection and segmentation of signal/image are important operations in the general fields of image processing and computer vision. Here, we present a stochastic FitzHugh-Nagumo mechanism which utilizes activation-excursion process and show how it can be used to address various problems in noisy environment, including edge detection and signal/image segmentation. The stochastic nature of our method makes it robust against noise, especially for handling poor contrast and highly noise polluted signal/image. We demonstrate the superiority of our method against the recently published deterministic FHN and some conventional methods on a few synthetic and real signals/images.

Key words: *edge detection, segmentation, stochastic FitzHugh-Nagumo system, deterministic FitzHugh-Nagumo model, activation-excursion process, stochastic resonance.*

Introduction

The response of dynamical systems to noise has long been an active field of study, mostly driven by its enormous relevance in numerous applications in engineering, physics, biology, and medicine [1, 2]. Often, noise is an undesirable element of the dynamical system and considerable previous work has focused on techniques that can suppress its effects in real applications. However, not all noise is bad; indeed, sometimes the system ‘‘tunes’’ itself to achieve optimal response as a function of a given noise floor.

This has led to extensive investigations of noise-mediated cooperative behavior, e.g., stochastic resonance [3], and noise-enhanced propagation, as well as more rigorous investigations into the behavior of bifurcating dynamical systems in the presence of noise [4]. In this paper, we investigate the effect of noise and periodic signal driving in the FitzHugh-Nagumo model (FHN) then we call it stochastic FHN which has become a popular representation of dynamical systems for several reasons. First, its relative simple structure sometimes allows one to make analytical progress. Second, by varying the parameters, the FHN admits a number of standard dynamics including periodic oscillations, stable fixed points, and excitability. Third, the FHN has been used as a simple model for both neurons and cardiac tissue, making it relevant to biomedical systems. Our main approach consists of observing the activation-excursion process of the stochastic FHN equations. We will do this for both the monostable and for bistable system. In the latter case, the stochastic FHN allows us to investigate the improvement of performance than the old one (determenistic FHN).

The Fitzhugh-Nagumo [5] equations are a set of simple equations which exhibit the qualitative behaviour observed in neurons, namely: quiescence, excitability and periodic behaviour. The form we will use here

$$\frac{\partial u}{\partial t} = D_e \nabla^2 u + \frac{1}{\varepsilon} f(u,v), \qquad \frac{\partial v}{\partial t} = D_i \nabla^2 v + g(u,v) \tag{2.1}$$

where D_e and D_i are diffusion coefficients taken to be nonnegative for the variables u and v respectively, the parameter ε is a positive small constant $(0 < \varepsilon << 1)$, and $f(u,v)$ and $g(u,v)$ are reaction terms taking the forms:

$$f(u,v) = u(1-u)(u-a) - v, \qquad g(u,v) = u - bv \tag{2.2}$$

$f(u,v)$ is a cubic function and assumed to be smooth enough. Parameters a and b are positive constants.

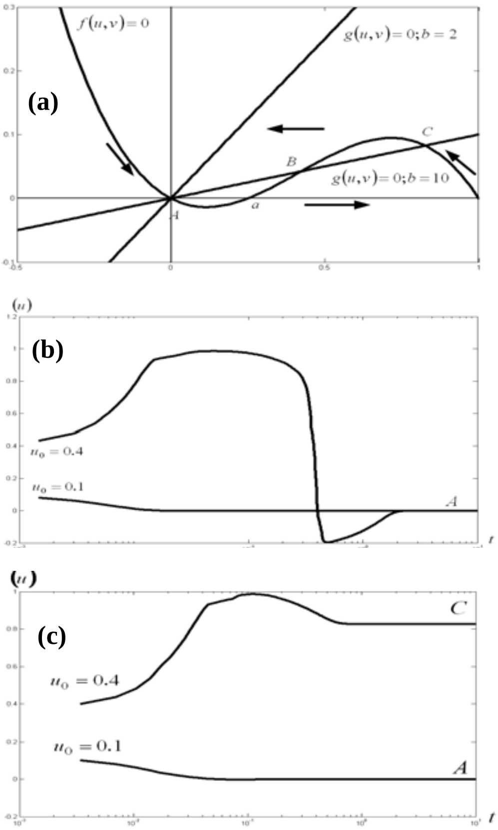

Figure 1. (a) Trajectories of determininstic FHN nullclines in the phase plane. The three nullclines depicted are $f(u,v)=0$, $g(u,v)=0$ with $b=2$ and $g(u,v)=0$ with $b=10$. The steady state A and C are stable; whereas B is unstable. Arrows show the set of solutions (u,v) trace when the system escapes from stable point. (b) The log-scaled of temporal response of two steady state solutions of *u* component where intial conditions are set different. Setting b=2, the system turns to monostable system showing two intial conditions converge at the single steady state A. (c) Otherwise setting b=10, the system is to be bistable system where each initial condition goes toward either of two steady state A or C. The parameters used here are $a=0.25$, $\varepsilon=10^{-4}$, $b=2$ or $b=10$.

The analysis of the deterministic FHN model has shown that depending on the level of stimulation there exist three regimes—an excitable, a bistable, and an oscillating one. In the excitable state, the system stabilizes at a resting state. Small displacements from this state are followed by a rapid return; large ones evoke a different response (the action potential) before the system returns back to rest. In the oscillating regime, the FHN model displays periodic oscillations corresponding to a limit cycle in its phase space. Finally, the bistable region corresponds to the transition between one stable point to another. In the bistable region, the FHN either switches to a resting state A or C depending on the initial condition (see Fig. 1). When the system is monostable, it describes a uniform stable steady state. The large phase difference between two sets of solutions generates an impulse at their boundary. This implies that it is applicable to edge enhancer. Whereas if the model is bistable, the set of diffusion-less differential equations in Eqs. (2.1) seperates the initial conditions into two different stable steady state where the two nullclines are crossing. This phenomenon is useful for segmentation.

Stochastic Excitable FHN As Edge Enhancher

We propose the representation of the stochastic FHN model is given by

$$\begin{aligned} \frac{\partial u}{\partial t} &= D_e \nabla^2 u + \frac{1}{\varepsilon} f(u,v), \\ \frac{\partial v}{\partial t} &= D_i \nabla^2 v + g(u,v) + s + \sqrt{2D}\xi \end{aligned} \tag{3.1}$$

where *f*(*u*,*v*) and *g*(*u*,*v*) are reaction terms as defined in Eqs (2.2). The one dimensional version variables $u(x,t)$ and $v(x,t)$ are an activator and an inhibitor variable, respectively. $s(x)=A_0\cos(\omega x+\phi)$ is a periodic signal and $\sqrt{2D}\xi(x,t)$ is a white Gaussian noise with intensity *D*.

In Fig. 3b, however while the contrast is very poor (SNR=10dB), deterministic FHN is not a sophisticated tool to make the edges isolated since some unimportant edges are generated. Edges detected by coherence FHN look much better. The edges are partly localized since one pulse in between is preserved (see Fig. 3c). This result is due to the fact that the firing process seems not strong enough to perturb the *v* nullcline for eliminating the pulse. An approach to cope with this problem is adding both noise and a weak periodic signal instead of gaussian noise alone to evoke firing. The result in Fig. 3d indicates that the edge location could be identified by the use of the proposed stochastic FHN.

In order to evaluate their generalization ability, we applied the edge enhancers/detectors to noisy image shown in Fig. 4a. Fig. 4b shows the ideal edges made from an original noiseless image by application of the canny detector. The well-known Canny’s operator is based on three criteria: good detection, localization, and the uniqueness of the response per edge. They are the main reason why the resulting edges from noisless image considered as ideal edges. In the edges enhanced by Canny and Sobel Filter from noisy image can be seen in Fig 4c and 6d respectively, the edges contain much noise and appear discontinuous.

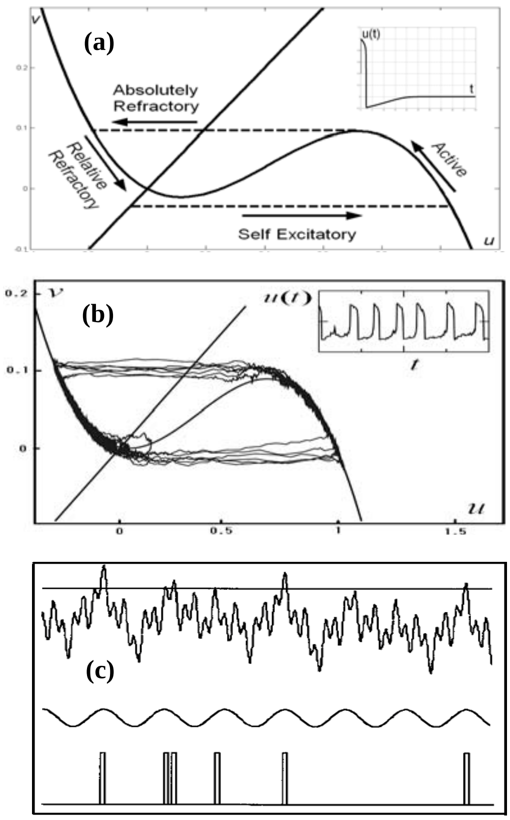

Figure 2 (*a*) Nullclines of the FHN system (thin solid lines) and a deterministic trajectory (D = 0; *s(x)* ≡ 0) in the phase plane. The inset shows the time evolution of the *u* variable. (*b*) A stochastic realization for D = 0.03, *s(x)* ≡ 0. Noise-driven excursions through the phase plane imply a spike train in the variable *u*(t), resembling the spontaneous electric activity of a neuron. For both panels *b*=2, a=0.25, $\varepsilon = 0.0001$ and diffusion-less of Eq. (3.1) are utilized. (c) Illustration of threshold crossing mechanishm to evoke ***firing*** in FHN system. The top trace shows the sum of random and periodic terms; the horizontal line indicates the fixed activation threshold. The middle trace shows, for comparison, the periodic part of the input (*s*(x)). Each time the total input crosses the threshold on the way up, a unit pulse is generated in the output (lower trace).

In the edges enhanced by deterministic FHN, the noise is enhanced by mistake forming small spikes or artifacts causing the edges could not be completely identified. In contrast, in the edges enhanced by the stochastic FHN, there is less noise at edges. The enhanced edges are continuous and accurately localized. It is qualitatively or visually even better than those edges of noiseless image yielded by Canny detector. As predicted, this result shows the performance of edge detection is higher than that of some edge detectors. This attempt is aimed to prove that the cooperative phenomenon of noise and periodic signal is useful for extracting 2D edges.

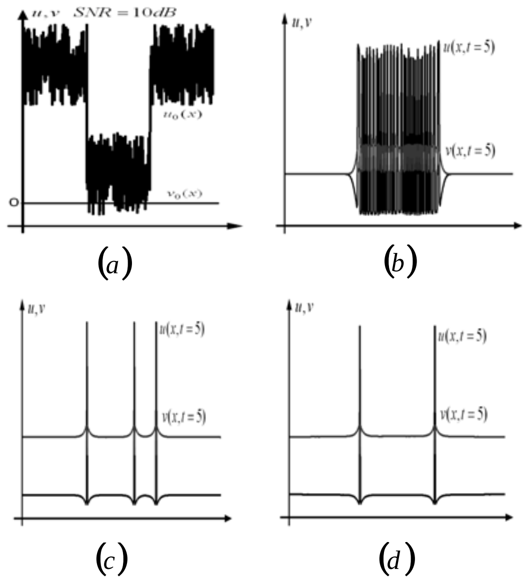

Figure 3. The numerical proof of periodic threshold causing the higher probability of the monostable system to ***fire*** yields the better edge of signal feating both deterministic FHN and CR (coherence resonance) FHN. Clock-wise: (a). Noisy signal (b). Edges enhanced by deterministic FHN (c). Edges detected by CR FHN (d). Edges enhanced by SR FHN. The utilized parameters here are similar to Figure 1b. Additional parameters:

(b). $D_e = 1; D_i = 6; D = 0$.

(c). $D_e = 1;\ D_i = 6;\ D = 0.125$.

(d). $D_e = 1;\ D_i = 6;\ D = 0.125; A_0 = 0.1,\ \omega = 0.5, \phi = 0$.

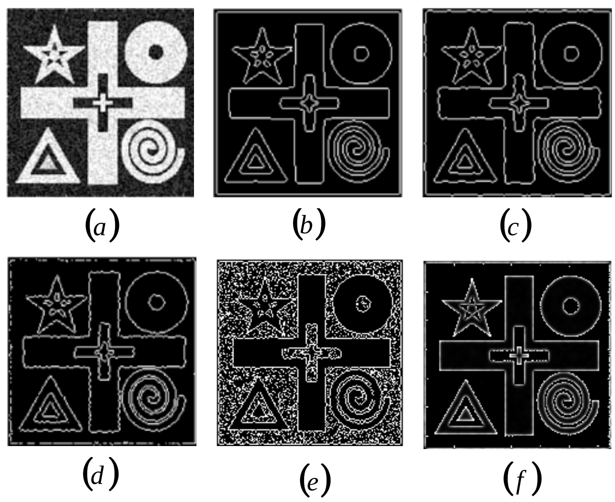

Figure 4. (a). Noisy Artificial Image (b). Result of applying Canny Detector on noiseless image. (c). Canny detector for noisy image (d) Sobel Filter. (e).Deterministic FHN (f).Proposed Stochatis FHN.

Stochastic Bistable FHN for Image Segmentation

The resulting general features of SR in two-stable system are similar to the single-state case. One difference is that now the optimal condition occurs when the system switches about twice per drive period rather than once. Such an effect is apparent even when the perturbation is weak enough not to appreciably affect the noise-induced switch process. However, the interplay of inherent noise and periodic driving mechanism results in a sharp enhancement of segmented signal/image.

our approach turns out to achieve very appealing performance with respect to both segmentation and blurring quality criterion. Stochastic FHN are design to perform in noisy environment. It produces enhanced segments while simultaneously removes the noise (see Fig. 5 and Fig. 6). It paves the way to more reliable segmentation method and outperforms its counterparts.

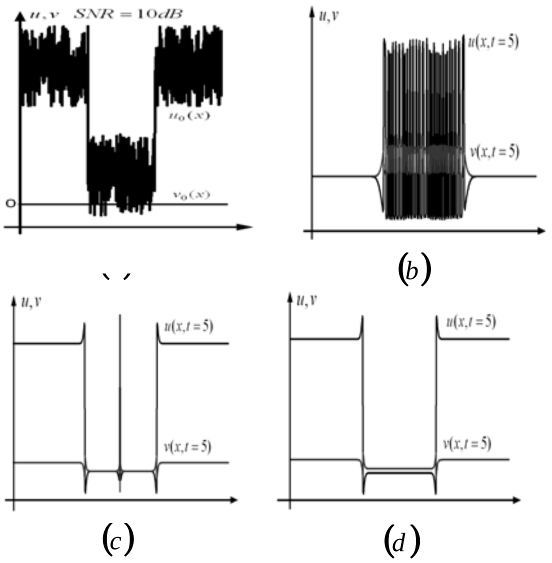

Figure 5. The numerical proof of periodic threshold causing the higher probability of the bistable system to ***fire*** yields the better signal segmentation superior to both deterministic FHN and CR (coherence resonance) FHN. In this case, due to the SNR is given quite small, the amplitude of periodic signal must be set to be significantly large in order for evoking the system to fire. The cooperative phenomenon of both gaussian noise and periodic signal successfully vannish the unnecessary pulse(s) seperating the signal pretty good. The utilized parameters here are similar to Figure 1c. Additional parameters:

(b). $D_e = 1; D_i = 6; D = 0$.

(c). $D_e = 1;\ D_i = 6;\ D = 0.125$.

(d). $D_e = 1;\ D_i = 6;\ D = 0.125; A_0 = 0, \omega = 0.5, \phi = 0$

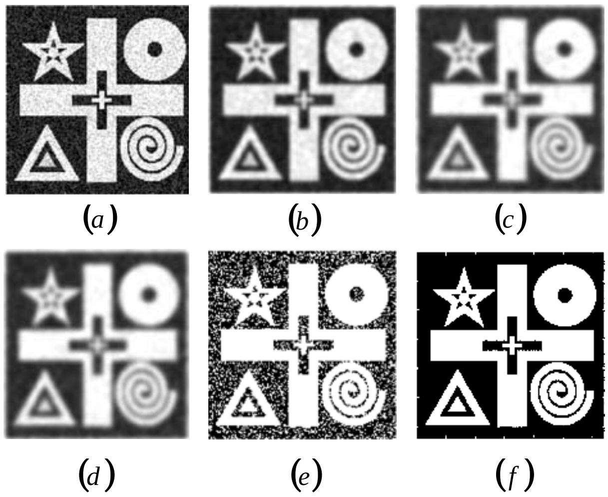

Figure 6. Separation of heteregenous objects from noisy background.. Clock-wise: (a) Syntethic 156x156 noisy image. The brightness values are uniformly distributed between 0 and 0.5 for the background, between 0 and 0.75 for the triangle, circle, star, spiral and the cross. (b). Perona-Malik Anisotropic Diffusion. (c). Forward and Backward Diffusion. (d) Complex Diffusion (d). Deterministic FHN (Parameters used are similar to Fig 10b). (e). Stochastic FHN (Parameters utilized are similar to Fig 5d). Note: (b) (c) and (d) obtained after 50 iterations.

References

Bruce Mcnamara, & Kurt Wiesenfeld (1989). Theory of Stochastic Resonance," *Physical Review A*, **39**, 4854-4869.

Luca Gammaitoni, Peter Hanggi, Peter Jung, Fabio Marchesoni (1998). Stochastic resonance. *Reviews of Modern Physics*. **70**, 223-287.

H. Haken (1975). Cooperative phenomena in systems far from thermal equilibrium and in nonphysical systems. *Reviews of Modern Physics*, **47**, 67-121.

B. Lindner, & L. Schimansky-Geier (1999). Analytical approach to the stochastic FitzHugh-Nagumo system and coherence resonance. *Physical Review E*, **60**, 7270-7276.

Atsushi Nomura, et, al (2003). Realizing Visual Functions with the Reaction-Diffusion Mechanism. *Journal of the Physical Society of Japan*, **72**, 2385-2395.

A. M. Turing (1952). *Philos. Trans. R. Soc. London*, Ser. B, **237**.

P. Perona, & J. Malik (1990). Scale-Space and Edge Detection Using Anisotropic Diffusion. *IEEE Trans. Pattern Anal. Mach. Intell.*, **12**, 629-639.

G. Gilboa, N. Sochen, Y. Y. Zeevi (2004). Image Enhancement and Denoising by Complex Diffusion Processes. *IEEE Trans. Pattern Anal. Mach. Intell.*, **26**, 1020-1036.